DOCUMENT RESUME

ED 476 617 SE 067 886

TITLE	Getting Students Ready for Algebra I: What Middle Grades Students Need To Know and Be Able To Do.
INSTITUTION	Southern Regional Education Board, Atlanta, GA.
PUB DATE	2003-00-00
NOTE	45p.
AVAILABLE FROM	Southern Regional Education Board, 592 10th Street NW, Atlanta, GA 30318; tel: 404-875-9211; web site: www.sreb.org
PUB TYPE	Books (010) -- Reports - Descriptive (141)
EDRS PRICE	EDRS Price MF01/PC02 Plus Postage.
DESCRIPTORS	*Algebra; *Competency Based Education; Curriculum Development; Educational Quality; Evaluation Criteria; *Instructional Effectiveness; Mathematics Education; Middle Schools

ABSTRACT

This report is designed to assist curriculum planners, principals, and teachers intending to help develop frameworks, course syllabi, lesson plans, assignments, and staff development activities that will enable students to meet the demands of high-level mathematics courses. Using the National Assessment of Educational Progress (NAEP) as a reference, definitions of basic, proficient, and advanced levels of proficiency were used to evaluate items on the middle grades assessment to define what students should know and be able to do to be successful in Algebra I. (KHR)

Reproductions supplied by EDRS are the best that can be made
from the original document.

ED 476 617

SE SETR 0123

SREB

High Schools That Work

Making Middle Grades Work

Making Schools Work

Getting Students Ready for Algebra I:

What Middle Grades Students Need to Know and Be Able to Do

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

M. Sullivan

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

1

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

This document has been reproduced as received from the person or organization originating it.

Minor changes have been made to improve reproduction quality.

Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

BEST COPY AVAILABLE

Southern Regional Education Board

592 10th St. N.W.
Atlanta, GA 30318
(404) 875-9211
www.sreb.org

CR 806 880

Foreword

Visits in the last several years to well over 100 middle grades schools indicate that the schools' goals and priorities often are unclear to teachers, students and the community. States have set content and performance standards in each core academic area, but these standards need to be translated into daily work in the classroom. Identifying readiness indicators for Algebra I is one way to translate middle school mathematics content standards for the classroom. **It is also a way to guide high schools in planning and implementing catch-up courses in mathematics for incoming ninth-graders**. The Southern Regional Education Board (SREB) worked with a panel of teachers and experts from the Educational Testing Service (ETS) to develop Algebra I readiness indicators.

Based on their objective judgments, the panel members — using the National Assessment of Educational Progress (NAEP) as a reference — developed definitions of the Basic, Proficient and Advanced levels of proficiency. The panel used these definitions to evaluate items on the Middle Grades Assessment — which uses publicly-released items from NAEP — and to define what students should know and be able to do to be successful in Algebra I. The SREB panel used its definitions of Basic, Proficient and Advanced to determine the proficiency level for each item in the Benchmark Proficiency Progression charts, the Learning Activities and Applications, and the Proficiency Level Illustrations in this report. The process the SREB panel used was less complex than the process NAEP uses. Therefore, **the panel's determinations of the proficiency levels are not to be construed as equivalent to the NAEP process.**

This report is not intended to answer all curriculum-related questions or to serve as a complete teaching plan. Instead it is designed to assist curriculum planners, principals and teachers. It is intended to help them develop frameworks, course syllabi, lesson plans, assignments, assessments and staff development activities that will enable students to meet the demands of the high-level mathematics courses they will encounter after leaving the middle grades. **This report is a tool to help middle grades and high schools set goals and priorities for mathematics that will get all students ready for Algebra I and help them complete it successfully by the end of ninth grade.**

States that are a part of SREB are committed to reaching every student through a series of 12 goals designed to lead the nation in educational achievement. One of these 12 goals specifically addresses the middle grades:

Achievement in the middle grades for all groups of students exceeds national averages and performance gaps are closed.

In the 2002 *High Schools That Work* (*HSTW*) survey, 32 percent of career/technical students reported taking some form of Algebra I by the end of grade eight. According to the NAEP 2000 Mathematics Assessment, about 25 percent of eighth-graders nationally took Algebra I.[1] Two-thirds of the nation's eighth-grade students are at or above the Basic level in mathematics as measured by NAEP, but on NAEP's most recent assessment, students as a whole from only four SREB states performed at that level. One-fourth of eighth-graders nationally are at or above the Proficient level in mathematics as measured by NAEP, but students as a whole from only three SREB states performed at that level. Based on the most recent NAEP state assessment in mathematics, in SREB states using Basic as the level of readiness for Algebra I, the percentages of students ready for Algebra I range from 41 percent in Mississippi to 70 percent in North Carolina. In SREB states using Proficient as the level of readiness for Algebra I, the percentages of students range from nine percent in Mississippi to 36 percent in North Carolina.

This report provides guidance for a rigorous mathematics program in the middle grades and for catch-up mathematics courses at the high school level. High quality mathematics instruction based on a solid set of essential standards can prepare all students to perform at least at the Basic level of proficiency and an increasing percentage of students to perform at the Proficient level.

Gene Bottoms
Senior Vice President
Southern Regional Education Board

[1] National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2000 Mathematics Assessment at http://nces.ed.gov/nationsreportcard/mathematics/results.

Why Develop Readiness Indicators for Algebra I? 1
How Were the Readiness Indicators Developed? 2
How the Report Is Organized 3
How Can Educators Use this Report? 4
Overarching Mathematics Skills and Concepts 5

Process Readiness Indicators
1. Problem Solving 6
2. Reading and Communicating 7
3. Estimating and Verifying Answers and Solutions 8
4. Logical Reasoning 9
5. Using Technology 10

Content-specific Readiness Indicators 11
Content-specific Readiness Indicator 1. 12
Proficiency Level Illustrations for Content Indicator 1 13
Content-specific Readiness Indicator 2. 14
Proficiency Level Illustrations for Content Indicator 2 15
Content-specific Readiness Indicator 3. 16
Proficiency Level Illustrations for Content Indicator 3 17
Content-specific Readiness Indicator 4. 18
Proficiency Level Illustrations for Content Indicator 4 19
Content-specific Readiness Indicator 5. 20
Proficiency Level Illustrations for Content Indicator 5 21
Content-specific Readiness Indicator 6. 22
Proficiency Level Illustrations for Content Indicator 6 23
Content-specific Readiness Indicator 7. 24
Proficiency Level Illustrations for Content Indicator 7 25
Content-specific Readiness Indicator 8. 26
Proficiency Level Illustrations for Content Indicator 8 27
Content-specific Readiness Indicator 9. 30
Proficiency Level Illustrations for Content Indicator 9 31
Content-specific Readiness Indicator 10. 32
Proficiency Level Illustrations for Content Indicator 10 33
Content-specific Readiness Indicator 11. 34
Proficiency Level Illustrations for Content Indicator 11 35
Content-specific Readiness Indicator 12. 36
Proficiency Level Illustrations for Content Indicator 12 37

Answers to Proficiency Level Illustrations 38

SREB Performance Descriptors for Grade Eight Mathematics

Basic

Students performing at the Basic level are able to work with the four arithmetic operations in one- or two-step word problems. They can identify and apply some mathematical definitions on an elementary level. While these students are likely to possess a satisfactory level of competency with computation, they routinely lack a conceptual understanding of many fundamental mathematical concepts and are usually not able to regularly implement simple reasoning and problem-solving strategies.

At the Basic level, students are able to recognize pictorial representations of fractions, read rulers and scales, and recognize which units of measurement are most appropriate for a given situation. They can identify geometric shapes and properties of those shapes and can visualize transformations of figures. Students at this level are able to construct graphs, such as bar graphs and pictographs if a scale is given in the problem, and can read and interpret information from graphs. They also are able to work with simple probabilities and can find the mean of a set of numbers. In algebra, students at the Basic level can extend simple number patterns, work with positive and negative numbers, evaluate simple expressions, and solve equations with one variable. They are beginning to develop an understanding of representation by locating ordered pairs on a coordinate grid and by constructing number sentences.

Proficient

Students performing at the Proficient level possess a working knowledge of many fundamental mathematical ideas and are beginning to interpret and apply concepts and abstract ideas. They are able to work with problems containing more than one or two pieces of mathematical information. These students generally exhibit an emerging knowledge and understanding of more formalized algebra topics.

At the Proficient level, students are able to use reasoning in their numerical computations and in working with data in order to interpret their results in the context of the problem. For example, in a problem requiring students to find the number of buses needed for a field trip, if the result is 12 1/4 buses, students will know that 13 buses will be needed for the trip. They can also extract information from graphs and combine that information with their knowledge of other topics in mathematics to solve a problem. At this level, students are able to work with measurement topics that incorporate several ideas. For instance, they are able to recall that the sum of the three angles of a triangle is 180 degrees and can apply that information correctly in a problem. In the area of algebra, students can work with representations to perform operations, such as combining like algebraic terms and solving linear equations in two variables, and they may be beginning to develop an understanding of algebraic identities.

Advanced

Students performing at the Advanced level are able to work confidently with abstract representations of fundamental mathematical concepts. They can work effectively with whole numbers, integers, rational numbers and their equivalents. These students are developing mathematical reasoning processes and analytical techniques in order to solve more complex problems, and they may be able to use a more efficient solution strategy if one is available.

At the Advanced level, students are able to utilize properties in geometry to analyze geometric situations and can begin to recognize the formal structure of geometry. For example, they are able to identify counterexamples for certain properties of geometric shapes. In algebra, students at the Advanced level possess a thorough understanding of patterns, such as the ability to generalize patterns, construct algebraic representations of patterns and work with complex patterns involving multiple operations that may include powers. These students may also be successful in solving some types of non-routine problems.

Why Develop Readiness Indicators for Algebra I?

With state accountability programs now operating or scheduled in every SREB state, states, districts and schools have never had a more compelling need to raise students' mathematics achievement — especially in the middle grades — than they do now. The *No Child Left Behind Act of 2001* gives schools a further incentive to improve mathematics achievement. They are required to work toward getting all groups of students to perform at least at the states' defined levels of Proficient in mathematics.

As more states increase their high school graduation requirements in mathematics and as the number of students in these states who cannot pass the state algebra test increases, more students will not receive high school diplomas. Algebra I is the key — and the barrier — to students' ability to complete a challenging mathematics curriculum in high school. In the 2001 ninth grade follow-up study, 62 percent of students in *HSTW* sites who took something called "algebra" in the middle grades took college-preparatory mathematics in grade nine. Of these students, 15 percent failed. In contrast, 35 percent of students who did *not* take something called "algebra" in the middle grades took college-preparatory mathematics in grade nine. Of this group, 27 percent failed. Clearly, students introduced to algebraic skills and concepts earlier are more likely to succeed in a high school algebra course if they are given the opportunity to do so.

Also according to the 2001 ninth grade follow-up study, 39 percent of students in grade eight who scored in the *two lowest quartiles* on the Making Middle Grades Work (MMGW) mathematics assessment and who took college-preparatory mathematics in grade nine made a grade of "D" or "F." Of the students scoring in these same two lowest quartiles in achievement who took lower-level mathematics in grade nine, 39 percent made a "D" or an "F." **This means that almost two out of five eighth-graders scoring in the lowest two quartiles in mathematics achievement *fail* whatever mathematics course they take in grade nine. However, when these students are enrolled in higher level courses, they are *not* more likely to fail.** Given quality teaching, extra time and help, even more of these students could succeed.

Greater percentages of high school graduates are enrolling in postsecondary education programs that require completion of higher levels of mathematics than many schools currently require for graduation. According to the 2002 *HSTW* survey, 56 percent of career/technical students completed four or more mathematics courses during high school. This means that 44 percent did not. According to SREB's *Benchmark 2000* series, in every SREB state more than 20 percent of students entering two-year colleges needed remedial courses in mathematics.[2] The highest remediation rate was more than 70 percent.

A score of 19 is a common standard on the ACT for placement in remedial courses. In the SREB states where most high school seniors take the ACT, the percentages of students who scored below 19 in mathematics ranged from 49 percent to 74 percent. Nationally, 48 percent of students scored below 19 in mathematics on the ACT. For four-year colleges and universities, the percentages of entering students who took at least one remedial course in reading, writing or mathematics ranged from 10 percent to more than 40 percent. **Clearly, completing a challenging mathematics curriculum is essential to being prepared for postsecondary education.**

Obtaining and succeeding in the "good" jobs of today's economy require an ever-increasing breadth and depth of mathematical skills and concepts. According to *The Skills Gap 2001* by the National Association of Manufacturers, "Poor reading, writing, math and communication skills were significant concerns."[3] Twenty-six percent of respondents cited that one of the most serious deficiencies in current employees was inadequate mathematics skills. According to *Reality Check 2002*,[4] 63 percent of employers give students fair or poor ratings in mathematics. These findings support the conclusion that success in college-preparatory mathematics courses in high school leads to more opportunities after high school. This is not very likely to occur without a correspondingly rigorous mathematics curriculum in the middle grades. **This middle grades curriculum must help students complete Algebra I in grade eight or prepare for Algebra I in grade nine.**

In many middle grades classrooms, teachers are unsure how to prepare students for Algebra I. They are teaching mathematics without aligning what they teach to a rigorous mathematics framework aimed at getting students ready for Algebra I. There are numerous curriculum documents at the national, state and district levels that provide standards and frameworks regarding what students should learn in middle grades mathematics. However, these documents give teachers little or no guidance about the depth of understanding students need on the most essential readiness indicators for Algebra I. **As a consequence, far too many middle grades teachers repeat sixth-grade mathematics content in grades seven and eight rather than focusing on developing students' understanding of these essential readiness indicators.**

[2] *Reducing Remedial Education: What Progress Are States Making?*, SREB Educational Benchmark 2000 Series.
[3] Center for Workforce Success and Andersen, *The Skills Gap 2001*, National Association of Manufacturers, 2001.
[4] *Reality Check 2002*, Public Agenda at www.publicagenda.org.

BEST COPY AVAILABLE

How Were the Readiness Indicators Developed?

This report is based on a review of current state and district curriculum guides in mathematics and discussions with a panel of curriculum developers and expert teachers of algebra and higher-level mathematics.

Any attempt to build a rigorous middle grades mathematics curriculum must begin with these two questions:

- **What are the essential mathematics skills and concepts (Readiness Indicators) that students must master by the end of the middle grades, or in a catch-up course in grade nine, in order to succeed in Algebra I?**
- **What are some examples of the proficiency levels for each of the mathematics Readiness Indicators identified?**

Preparation of this report began with a review of major mathematics curriculum documents including:

- the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics*,
- curriculum materials underlying the National Assessment of Educational Progress (NAEP), and
- curriculum guides from the SREB Middle Grades Consortium member states and selected other states.

An analysis of these frameworks yielded a fairly lengthy list of standards for middle grades mathematics. An expert panel of classroom teachers and curriculum specialists selected from this list those standards that represent the most essential skills and concepts for success in Algebra I. Identifying curriculum standards that are most important for high school success ensures that essential material does not fall through the cracks — either for lack of time or because it is too challenging. To help panel members narrow their selections, they were asked to think about the following questions:

- What knowledge, skills and experiences in mathematics do students entering high school most often lack that they should have acquired in earlier grades?
- What skills, knowledge and experiences in mathematics separate students who enter and succeed in a rigorous high school mathematics curriculum from those who do not?
- What skills and knowledge in mathematics that students should acquire in the middle grades take the most time to reteach?
- What deficiencies in knowledge and understanding are most difficult to remedy and which ones continue to plague students in later courses?

After submitting their responses, panel members met and began discussions to identify the set of essential mathematics skills and concepts. After numerous telephone conferences and e-mail communications, the panel reached a consensus regarding the essential mathematics skills and concepts for success in Algebra I — the 17 Readiness Indicators in this report. Then, with the help of ETS and SREB staff, the panel developed definitions of Basic, Proficient and Advanced levels of mathematics performance at grade eight. (See page iii.) These definitions guided the panel and staff to develop the Benchmark Proficiency Progression charts for each Readiness Indicator and to create the Learning Activities and Applications. Panelists reviewed various assessment items including the publicly-released NAEP items and made judgments about which items best illustrated their definitions of the proficiency levels. These judgments are *not* to be construed as equivalent to the NAEP standards of Basic, Proficient and Advanced. The panel also created test items to supplement the publicly-released test-item examples at each proficiency level. Then panel members and other mathematics experts reviewed the final draft of the report and offered suggestions for making it useful to educators.

How the Report Is Organized

This report is organized around 17 Readiness Indicators; the first set of five Process Readiness Indicators represents the skills and concepts that should be incorporated into mathematics at all grade levels and in all courses. The second set of 12 Readiness Indicators addresses essential content-specific skills and concepts that prepare students for Algebra I. Each Readiness Indicator is described and includes some examples of how the indicator relates to the preparation for algebra, as well as suggestions to help teachers teach the skills and concepts that students need.

A Benchmark Proficiency Progression chart listing major benchmarks for the indicator at each proficiency level follows each description. These charts help teachers and administrators identify the skills and concepts that students should master to succeed in Algebra I. While many students are performing below the Basic level in mathematics achievement, the Making Middle Grades Work (MMGW) goal is to get *all* students to achieve at the Basic level and an increasing percentage to achieve at the Proficient and Advanced levels. The MMGW initiative believes that under these conditions, students with quality teaching and quality extra help and time will be successful in Algebra I.

Next is a list of Learning Activities and Applications that represent suggestions for Basic, Proficient and Advanced assignments. These activities connect the mathematics Readiness Indicators to mathematics contexts in other subjects, to other areas of mathematics and to the world outside school. All of these activities and applications encompass one or more of the first five Process Readiness Indicators — the Overarching Skills and Concepts. Extending mathematics instruction beyond just drill-type exercises is necessary, not only to generate student interest, increase motivation and challenge students intellectually, but also to help students realize the importance of mathematics in their everyday lives and in their futures. This type of instruction also provides a context of understanding and retaining mathematical knowledge. **The goal of promoting numeracy across the curriculum should be shared by *all* teachers.**

The section containing the 12 Content-specific Readiness Indicators also provides test-item examples called Proficiency Level Illustrations. The sample test items show how each Content-specific Readiness Indicator translates into test items at each level of proficiency.[5] Many of the examples are publicly released items from NAEP; others are publicly released items from individual state assessments or curriculum documents, and others have been created specifically by the panel for this report.

[5] All items were labeled at given proficiency levels by the panel members and **not** by the organizations associated with those assessments.

Benchmark Proficiency Progression

*Basic	*Proficient	*Advanced
Reached by all students somewhere in the beginning of the middle grades or somewhere around the point at which the topic is first introduced. Students at this level may succeed in Algebra I if given enough extra help and time.	Reached by more and more students at the end of the middle grades. Not all students will achieve this level in the middle grades, but all should be exposed to the knowledge, material and assignments associated with it. Students at this level are very likely to succeed in Algebra I.	Reached by some students by the end of the middle grades and by more students by the end of grade 10.

* These three proficiency levels are based on the categories used by the Making Middle Grades Work NAEP-based assessment of student progress. (See page iii.)

How Can Educators Use this Report?

The 17 Readiness Indicators can guide curriculum planners, principals and teachers in examining what they currently teach, how they teach it, and how much time they spend engaging students with each of these skills and concepts. They can compare the Readiness Indicators with the topics in their local curriculum frameworks and in their teaching plans. They should ask these types of questions:

- Are these essential topics in mathematics given the time and depth of coverage necessary to prepare students for Algebra I?
- Are there other topics that are given too much attention so that topics crucial for success in Algebra I are not fully developed or not included at all?

Educators can examine current standardized test results to see which items their students are not answering correctly. This analysis and a comparison to the Readiness Indicators provide information to help educators target content areas to improve student achievement and prepare them for Algebra I. The Benchmark Proficiency Progression charts guide educators to improve standardized test scores so that all students achieve at least at the Basic level and more and more achieve at the Proficient and Advanced levels. Without a clear and consistent understanding of what does and does not meet standards, it is impossible to claim that students have met the standards — even if they have only the most basic understanding of a topic.

The sample test items at each proficiency level can help curriculum planners, principals and teachers analyze how their students perform on the various indicators. Educators can gather a sample of their own assessments from mathematics classes to determine what proportion of their test items are below the Basic level, or at the Basic, Proficient or Advanced levels. **If students are not assessed in class at the Basic level or above, then they cannot be expected to perform at these levels on high-stakes tests.**

Curriculum planners, principals and teachers can use the sample Learning Activities and Applications at each proficiency level to help them evaluate student assignments. By collecting a sample of assignments around one or more of the readiness indicators, they can determine the levels of assignments given. For each indicator they should ask these types of questions:

- What is the most frequent level of assignments given? What percentage of assignments is at the Basic level or below? What percentage of assignments is at the Proficient level or above?
- As a result of looking at the level of assignments, have teachers been more purposeful in giving students higher level assignments?
- Are assignments limited to drill-type exercises that students have already mastered, or are they designed to have students practice new concepts and strategies to solve more complex problems?

Even students who are considered to be struggling need to be challenged by rigorous materials — especially open-ended problems. If they are not expected to do open-ended problems in class and for homework, then they cannot be expected to succeed on them on high-stakes assessments. **If students are never given Basic- and Proficient-level assignments and are not assessed at these levels or higher, then they cannot be expected to achieve at these levels.**

Educators need to examine how each of these indicators is taught.

- Are students assigned problems in real-world contexts that engage them in the learning process?
- Are the problems complex enough to force students to use logical reasoning and a variety of solution strategies and mathematics skills and concepts?
- Are students sharing their ideas for solving problems with their classmates? Are they explaining their solution processes to the class?
- Are students using technology to aid them in the solution process?

Finally, high school educators can use this report to develop tests that will help them determine the readiness indicators students have mastered and the ones they have not. To get ready for Algebra I, some students may require summer school while those with a greater deficiency may require an 18-week, specially designed catch-up course.

Students need to be challenged by interesting materials and provided the extra time and help they need to improve their achievement. Rather than experiencing low expectations by practicing skills they have already mastered, students should complete rigorous and challenging work that helps them meet and exceed higher expectations.

Overarching Mathematics Skills and Concepts

These five Readiness Indicators: problem solving, reading and communicating, estimating and verifying answers and solutions, logical reasoning, and using technology are processes found in all areas of mathematics. Whether in arithmetic, algebra, geometry or calculus, *problem-solving* skills are essential for analyzing problems and developing solution strategies. Students in the middle grades should have problem-solving experiences that promote mathematical learning and reasoning and that prepare them for Algebra I.

Integral to problem solving is *reading and communicating*. When faced with problems to solve in any area of mathematics, by the end of the middle grades students should have mastered reading the problem, interpreting what the problem states and understanding what is asked for in the problem. Students must be able to use the language of mathematics orally and in writing to explain the thinking processes, mathematical concepts and solution strategies they use in solving problems.

After solving any problem, students use a variety of techniques including *estimation to verify their answers*. The mastery of these techniques in the middle grades helps students recognize correct answers, check for reasonableness and identify their mistakes so that they can revise their work. After revising their work, students record their answers using appropriate units as required by the contexts of the problems. Sometimes the "answers" or solutions to problems are data sets, graphs or models. These "answers" also must be verified.

Logical reasoning is fundamental to all of mathematics. Middle grades students should have a variety of experiences that exemplify the types of reasoning they will encounter in their further study of mathematics and in other subject areas. Students, at least informally, should become familiar with examples of inductive and deductive reasoning.

Process Readiness Indicators:

1. Problem Solving
2. Reading and Communicating
3. Estimating and Verifying Answers and Solutions
4. Logical Reasoning
5. Using Technology

Finally, while *technology* is integral to our daily lives, it is especially important in the study of mathematics. Middle grades students should become proficient in the use of scientific calculators and graphing calculators to enhance their understanding of mathematical ideas and concepts. Technology supports the explorations of mathematical ideas and helps build understanding of these ideas and the use of inductive and deductive methods.

Students will not be able to successfully tackle real-world problems in a variety of contexts without middle grades teachers who incorporate these Overarching Skills in their mathematics classrooms. Since these skills promote higher-level thinking, they are valuable tools in developing an in-depth understanding of key mathematical concepts. Therefore, it is essential that these Overarching Skills not be seen as extras — that is, things to work on when time allows — but rather as processes that cut through all aspects of instruction and assessment to solidify understanding and increase retention of content-specific skills.

It should be noted that the skills and concepts listed in the Benchmark Proficiency Progression charts in this section are categorized by how the Overarching Skill is used, not by the mathematical content.

I hear and I forget.
I see and I remember.
I do and I understand.

Chinese proverb

1 Problem Solving

Problem solving is the context in which students apply mathematical skills and concepts to new situations. Problem-solving experiences enhance student understanding of the usefulness of mathematics and its power. In the real world most problems do not involve tricks, but real-world problems require a process to gain control over them so solutions can be explored and found. In 1957 George Polya added a reflection step to his earlier problem-solving phases which form the foundation of most self-teaching, problem-solving models in mathematics:

- Understand the problem.
- Make a plan.
- Carry out the plan.
- Review and discuss the complete solution.
- Look back and reflect on the solution and the processes.

Students who are prepared for Algebra I need to be able to use with confidence various problem-solving strategies, such as writing an equation, making a graph or table, choosing a formula, working backward, using guess-and-check, and finding more than one solution. But this problem-solving confidence also involves knowing which of the strategies or combination of strategies to use and when. As students tackle problems without obvious answers or solution strategies, they also must draw upon their own resourcefulness, organization and ability to think creatively. Problems that require applying their knowledge to new or unique contexts make their problem-solving skills more crucial and visible. These skills are only gained by continuous exposure to numerous multi-level problems in a variety of contexts.

While some students might claim that solving problems is the *only* thing they do in mathematics class, when used to describe a set of skills and strategies, problem solving means something more than filling in answers to word problems on a worksheet or test. Teaching problem solving in the middle grades requires consistent modeling of the process that students use to solve problems and the application of problem-solving strategies. Problem solving readily lends itself to cooperative learning groups as a way to encourage student thinking, experimenting and efforts to solve problems. By providing a variety of problem-solving experiences at the Basic, Proficient and Advanced levels, middle school students enhance their thinking skills and improve their abilities to tackle more difficult problems as they prepare for Algebra I.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ One- and two-step self-contained problems ▫ Problems with extraneous information ▫ Problems that require using a given formula	▫ Two-step problems with multiple solution strategies ▫ Problems with insufficient information ▫ Problems that require choosing the correct formula	▫ Multi-step problems with and without multiple solution strategies ▫ Non-routine problems ▫ Problems with no solution

Learning Activities and Applications

- Students discover formulas using hands-on materials. For example, the formula for the area and volume of more complex figures (e.g., trapezoids and cones), the formula for finding the total number of degrees inside a polygon, or combining formulas for the area of composite figures. Students write a formula then present it to the class. They describe their solution methods including the frequency with which they repeated the process to derive their formula. (Basic)
- Students determine the minimum number of pieces of drywall (which are manufactured in specific sizes) that are required to fully cover the walls of a room or inside an entire house. Students measure a room or rooms at home or at school. Then they record their steps and explain their answers. (Proficient)
- Students develop a scoring system for multiple-choice tests that discourages wild guessing, such as one that penalizes incorrect answers more than items not attempted. They explore how the system would affect different students, for example, a student who usually knows about two-thirds of the answers but guesses on the rest, and how the scoring system could be further modified so that students do not avoid challenging problems altogether. (Advanced)

2 Reading and Communicating

Of course general reading ability is a necessity in "doing" mathematics, but like any field, mathematics has its own specialized vocabulary. As students work at higher levels in mathematics and increase their understanding of mathematical concepts, their mathematical vocabulary must grow accordingly. Students must become fluent in the preciseness of language used in presenting mathematics problems, ideas and thinking. Skills in communicating mathematically include explaining the reasons for using particular solution strategies, demonstrating an understanding of concepts, defending ideas and thinking, and questioning others' thinking.

As students move through the middle grades and prepare for Algebra I, the presentations of their thinking and solution strategies for problems progress from informal narratives to more formal, sequential, and well-labeled presentations. These presentations include appropriate mathematical vocabulary and models, diagrams, graphs, and tables.

Part of teaching mathematical communication involves assessing students' understanding of mathematical vocabulary. Students who are unsuccessful in communicating their understanding of and solutions to problems can learn mathematical vocabulary just like they learn reading vocabulary. Vocabulary journals, mathematics journals and vocabulary word walls are just a few examples of how teachers can help students improve their skills in communicating mathematically. In the middle grades especially, students are reluctant to express themselves in any way that would lead to embarrassment. Teachers must encourage students to feel comfortable in expressing their thinking and strategies for solving problems openly and candidly.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Write and discuss brief descriptions of their solution strategies. ▫ Use cause and effect words (e.g., "if-then," "because") in describing reasoning.	▫ Ask appropriate questions regarding others' solutions. ▫ With some teacher guidance, write clear sequential descriptions of solution strategies, using appropriate vocabulary.	▫ Without teacher guidance, write multi-step explanations independently. ▫ Write hypotheses and conclusions that include the reasons for making them.

Learning Activities and Applications

- Students use straightedges to draw irregular polygons and then with rulers and protractors measure the sides and angles. Students describe the polygons for others to draw. Students ask questions and then compare the final drawings to the originals. (Basic)
- Students describe in their own words the steps to complete a task. For example, students describe ways to solve for a missing value in a proportion. Being prepared to revise their work, students write their descriptions and then share them orally with the class. (Basic)
- Students compare the costs of sending packages of different sizes and weights with several different carriers using rate charts or rate calculators found on the Internet. Students record their data in tables and after doing so, they write a brief summary conclusion about which service is better than the others and why. They present their findings to the class. (Proficient)
- Students read and discuss books about mathematics or with mathematics-related themes, such as *Flatland* by Edwin A. Abbott; *G is for Googol* by David M. Schwartz; and *Echoes for the Eye: Poems to Celebrate Patterns in Nature* by Barbara J. Esbensen. They independently prepare oral and written presentations describing how the content relates to particular topics in mathematics. (Advanced)

3 Estimating and Verifying Answers and Solutions

Sometimes students are so focused on carrying out a particular process that they do not step back from it to think about how to determine the reasonableness of their answers. Evaluating the reasonableness of an answer might strike some as something students should be able to do automatically. However students' inability to do so and the resulting mistakes are a common source of complaint among mathematics teachers. During the middle grades, students should begin developing an understanding that more complex problems sometimes involve making a decision about which form of an answer or solution is more useful, valuable or relevant. Students should become skilled in using appropriate representations of their answers and solutions, such as using various number forms or types of graphs. In these instances, students use other techniques to verify their work, such as spot checking the items in a data set, examining the accuracy of a scale used and values computed to create a graph, and analyzing a model to determine whether the model in fact represents what was asked for in the problem. Knowing when to solve problems mentally, recognizing when calculators are unnecessary and adjusting the precision of an answer are all skills that students should master in the middle grades.

When teachers observe and analyze particular patterns of mistakes in student work, they should not dismiss them simply as "careless," but should recognize that these mistakes stem from a lack of conceptual understanding of the mathematics at hand. More experiences with comparing, ordering, estimating and modeling can help students become more accurate in their work. Verifying answers and solutions to problems is one way for students to revise their work until it is correct and meets proficient or higher standards.

Benchmark Proficiency Progression

Basic

- Recognize when answers cannot be fractions or mixed numbers (e.g., numbers of people or buildings).
- Make front-end and rounding estimations.
- Verify that the units in the answer match units called for by the problem.
- Understand the magnitude of values and units in problems (e. g., the larger the unit the smaller the converted value).

Proficient

- Recognize when answers can and cannot be negative.
- Know when to use estimation strategies.
- Recognize shortcuts that can make computation easier and a calculator unnecessary.

Advanced

- Adjust the precision of answers based on the situation.
- Explain why there are no answers to a given problem.

Learning Activities and Applications

- Students identify large quantities for their classmates to estimate, such as the number of bricks in the side of a building, the distances certain animals can travel given their speed or the number of blades of grass on school property. Students then compare answers and solution methods and discuss the advantages and disadvantages of the different methods. (Basic)
- Students devise methods for calculating tips or sales tax mentally. They write explanations for their methods, using examples. They then present their methods to the class. The class compares the ease of the different methods. (Proficient)
- Provide students with measures of the sides of geometric figures such as:

  4 cm, 4 cm, 10 cm 5 cm, 6 cm, 13 cm 3 cm, 4 cm, 8 cm 2 cm, 2 cm, 6 cm 2 cm, 4 cm, 10 cm

  Students predict the type of geometric figure and its classification. Then they use rulers to draw each figure on centimeter graph paper and verify each others' work. Students experiment by altering the measures to obtain the geometric figures they predicted and record their results. Students then discuss their results, provide an explanation of what happened and determine a rule about triangles. (Advanced)

4 Logical Reasoning

The rules of logical reasoning that govern mathematics establish a valid means of manipulating with and concluding from mathematical statements. Logical reasoning includes formulating conclusions, constructing arguments and making conjectures based on observed regularities. As such, logical reasoning overlaps considerably with problem solving and communication.

The progression of students' thinking skills in the middle grades is tied closely to the depth of content. Students should leave the middle grades with sharper reasoning skills to evaluate conjectures, conclusions and arguments. They should have a basic understanding of the difference between inductive and deductive reasoning, as well as the ability to develop more sophisticated logical arguments to convince others, especially regarding problems in number theory, probability and geometry.

Students' logical reasoning abilities are best developed and assessed with open-ended problems or performance tasks where they must document their reasoning. Regardless of the topic, however, teachers must consistently develop students' understanding by challenging them to solve more complex problems and explain their reasoning to adequately prepare them for Algebra I.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Evaluate other students' solutions and solution strategies. ▫ Make general predictions (e.g., the outcome of events or the resulting areas or volumes from changes in dimensions).	▫ Make and evaluate basic logical arguments containing if-then (conditional statements), conjunctions, disjunctions and negations. ▫ Evaluate other students' reasoning.	▫ Complete simple truth tables. ▫ Identify statements of inductive and deductive reasoning.

Learning Activities and Applications

- Students make conjectures to answer the following questions and devise strategies to answer them. Which is more likely: typing your initials by hitting the keys on a computer keyboard at random or dialing your phone number by hitting the keys on a telephone at random? How long would you expect it to take to do each? Students collect data and look for regularities to support their predictions. (Basic)
- Recently, the president of a large grocery store chain has become concerned about the breakage rate of eggs on the store shelves. If a single egg is found broken on the shelves, the entire carton must be discarded. The rate is 1 broken egg every 13 cartons of a dozen eggs. One of the executives states that, without doing anything to reduce the number of broken eggs, the company could save money if it sold eggs in packages of 8 or 10 instead of 12. Students respond to this claim by listing factors to be considered and identifying different scenarios for when the statement would be true and when it would not. (Proficient)
- Students review statistical claims made in newspaper and magazine articles and in advertisements. They identify the ways in which data have been presented to convey a particular point of view and reasons why the claim might or might not be true. Students explain why or why not the displays or claims are misleading. (Proficient)
- As an example of inductive reasoning, students find the sums of several series of consecutive whole numbers, such as 1 to 7, 5 to 9, 10 to 30, and so on. Then they discuss their results and formulate a rule for finding the sum of a set of consecutive whole numbers. The activity can be extended to finding sums of consecutive integers. As an example of deductive reasoning, students use dot paper to apply Pick's Theorem (the sum of the number of dots on the sides of a figure divided in half and the number of dots inside a figure - 1 = the area) to investigate the areas of triangles, quadrilaterals, hexagons, and so on. They determine for which kinds of figures the formula is valid and verify their results using the traditional formulas for area. (Advanced)

5 Using Technology

Calculators and computers are indispensable tools in the fields of mathematics, science, technology and in everyday life. They permit searching for solutions to problems that without technology would be difficult because of the complexity or number of calculations involved. The study of mathematics and science requires students to use these technologies, but without an understanding of the underlying concepts and processes, their usefulness is limited. Technology should supplement and enhance learning, but it should never supplant it.

Technology needs to be an integral part of mathematics instruction throughout the middle grades. As students take on more complex calculations, create more complex explanations of solutions and reasoning, and explore different types of relationships, their technological skills must keep pace. As they experience solving problems in a variety of contexts with and without technology, they will develop an understanding of when technology is useful, when it is not useful and what its limitations are.

Teachers should guide students in the use of technology to enhance their learning, especially when graphing equations and inequalities, investigating properties of geometric figures, demonstrating and verifying theorems, and performing simulations in probability experiments.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Use a calculator to perform arithmetic operations. ▫ Use software to create simple tables.	▫ Use a scientific calculator. ▫ Use software to create spreadsheets that include totals and mean values. ▫ Use graphing software to create bar, line, and circle graphs.	▫ Use a graphing calculator to graph simple equations and functions and to find the mean and median of data. ▫ Use software to graph functions and explore other types of equations.

Learning Activities and Applications

- Students use 4-function calculators to determine how this type of calculator treats the order of operations. Then students use scientific calculators to repeat the activity and compare their results. Students share their conclusions with the class. (Basic)
- Students use a spreadsheet program to record outcomes of probability experiments or effects of changes in dimensions of regular figures upon perimeter, area and volume. They formulate conclusions and share them with the class. (Proficient)
- Students gather numerical data over time regarding a topic of their choice. They use graphing calculators to find the mean and median. Then they describe their data sets using the appropriate measure of central tendency and range. Students use graphing software to make line graphs of their data and describe any trends. (Advanced)

Content-specific Readiness Indicators

These 12 Content-specific Readiness Indicators define what the MMGW initiative believes are the essential content to prepare students for Algebra I. The order of topics in the list is not a teaching sequence, nor is it a ranking of topics from most important to least important.

1. **Read, write, compare, order and represent in a variety of forms: integers, fractions, decimals, percents, and numbers written in scientific and exponential form.**
2. **Compute (add, subtract, multiply and divide) fluently with integers, fractions, decimals, percents, and numbers written in scientific notation and exponential form, with and without technology.**
3. **Determine the greatest common factor, least common multiple and prime factorization of numbers.**
4. **Write and use ratios, rates and proportions to describe situations and solve problems.**
5. **Draw with appropriate tools and classify different types of geometric figures using their properties.**
6. **Measure length with appropriate tools and find perimeter, area, surface area and volume using appropriate units, techniques, formulas and levels of accuracy.**
7. **Understand and use the Pythagorean relationship to solve problems.**
8. **Gather, organize, display and interpret data.**
9. **Determine the number of ways events can occur and the associated probabilities.**
10. **Write, simplify and solve algebraic equations using substitution, the order of operations, the properties of operations and the properties of equality.**
11. **Represent, analyze, extend and generalize a variety of patterns.**
12. **Understand and represent functions algebraically and graphically.**

This section of the report contains the following for each indicator:

- explanation of how the indicator relates to success in Algebra I
- guidance for teaching the indicator
- Benchmark Proficiency Progression chart
- Proficiency Level Learning Activities and Applications
- Proficiency Level Test-item Illustrations

Most of the Proficiency Level Illustrations are from two outside sources, but some were written by the panel for this report. In most cases, the items from the outside sources appear as originally published, but in some cases, multiple-choice items have been converted to open-ended items or vice versa. These cases are noted. The two outside sources used are:

- The National Assessment of Education Progress (NAEP), Eighth Grade Mathematics Assessment, 1990, 1992, 1996, National Center for Education Statistics, U.S. Department of Education, http://nces.ed.gov/nationsreport-card; and
- Competency-Based Education Assessment Series: Mathematics, Ohio Council of Teachers of Mathematics (OCTM), Ohio Department of Education, Columbus, Ohio, 1997.

1 Read, write, compare, order and represent in a variety of forms: integers, fractions, decimals, percents, and numbers written in scientific and exponential notation.

Because much of students' future work in mathematics involves using various forms of the same number, it is imperative that by the time they leave the middle grades, students are able to convert quickly from one form to another. Students should readily be able to convert from percents to decimals to fractions and from numbers in scientific notation to numbers in standard form, and vice versa. Also, they should have mastery of the concepts of whole number and decimal place value, exponential notation, and graphing integers on a number line. Not only do these skills prepare students for the study of rational numbers and solving problems with very small and very large numbers in Algebra I, but they help them decide which solution or form of a solution to a problem is more useful, valuable or relevant. In the elementary grades, students spend a great deal of time learning about numbers and how to manipulate them; the focus in the middle grades must shift to using numbers to solve problems.

Teaching number concepts in the middle grades is important so that students can solve applications in algebra, for example. Teachers can use a variety of hands-on materials such as base-10 blocks, graph paper, 10-frames, fractions strips and fraction calculators to help students understand and master number concepts. Simple applications of very large and very small numbers can help engage students in learning number sense concepts so that they attain mastery.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Convert between fractions, decimals and percents. ▫ Estimate with fractions, decimals and percents. ▫ Use models to show fractions, decimals and percents. ▫ Compare and order fractions, decimals and percents. ▫ Understand the concept of integers.	▫ Compare and order integers. ▫ Write numbers in exponential and standard form. ▫ Write numbers in scientific notation. ▫ Find square roots of perfect squares.	▫ Compare and order rational and irrational numbers. ▫ Estimate square roots of non-perfect squares.

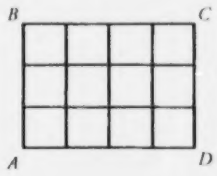
Learning Activities and Applications

- Students create and discuss situations and quantities that can and cannot be described by negative integers and then graph the values on a number line. They compare values in words and with symbols. (Basic)
- Students examine units of measure used for very large quantities and distances (light years, astronomical units) and very small quantities and distances (microns, nanoseconds). Then they write the measures in more familiar units, using scientific notation. (Proficient)
- Students find the lengths of the diagonals of squares of different sizes by drawing each on graph paper and measuring the diagonal. Next they use the Pythagorean relationship to find the lengths of the diagonals using estimation. Then they compare the results. (Advanced)

Proficiency Level Illustrations for Content Indicator 1

Basic

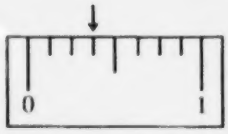
1-1 In the figure below, what fraction of rectangle *ABCD* is shaded? (NAEP)



(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

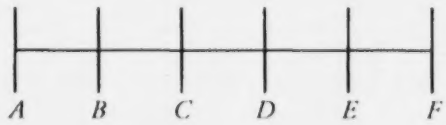
1-2 Which of the following represents the least amount? (OCTM)

(A) 0.3 (B) $\frac{2}{5}$ (C) 25% (D)



Proficient

1-3 The line segment *AF* is marked off into 5 equal parts. If you start at point *A* and go 77% of the way to point *F*, between which two letters will you be? (OCTM-multiple choice)



1-4 The average temperature for February is -2° C. Which of the following daily temperatures is closest to this average value? (OCTM)

(A) -10°C (B) -6°C (C) 1°C (D) 3°C

1-5 Write the following number using scientific notation: 23,456,000,000. (Panel)

Advanced

1-6 Name all the whole numbers whose square roots are between 5 and 6. (Panel)

1-7 The figure below shows the display on a scientific calculator. The value of the displayed number is between which of the following pairs of numbers? (NAEP)

4.752 E -02

(A) 0.04 and 0.05 (B) 0.4 and 0.5 (C) 4.0 and 5.0 (D) 40.0 and 50.0 (E) 400.0 and 500.0

BEST COPY AVAILABLE

2 Compute (add, subtract, multiply and divide) fluently with integers, fractions, decimals, percents, and numbers written in scientific notation and in exponential form, with and without technology.

During the middle grades, students should reinforce their mastery of computing with whole numbers, fractions, decimals and percents. To be ready for Algebra I, students also need to be able to compute with numbers in scientific notation and in exponential form. As students become proficient in computational skills, the emphasis should shift from drill-type exercises to one-step and two-step self-contained problems, and finally to multi-step problems in a variety of contexts. The skills students develop in translating words to numbers, operators and symbols will be increasingly important as they prepare for Algebra I.

If students enter the middle grades unable to perform the four arithmetic operations with whole numbers, fractions and decimals, then they should receive extra help and time outside of — but not instead of — their regular mathematics class to improve their skills. Also, they should know *when* to add, subtract, multiply or divide and to use inverse operations to check their answers. Using number lines, two-color counters and equation mats can help students learn how to compute with integers. To prepare students to use variables for unknown quantities (the essence of algebra), they need numerous experiences finding missing values in number frames and one-step equations in one variable in simple problem-solving contexts.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Perform computations in one- and two-step word problems. ▫ Model addition and subtraction of integers. ▫ Use the order of operations with whole numbers, fractions and decimals. ▫ Compute fluently with decimals, fractions and percentages.	▫ Compute fluently with integers. ▫ Perform computations in multi-step word problems. ▫ Compute with numbers in scientific and exponential forms with positive exponents. ▫ Use the properties of operations.	▫ Compute with numbers in scientific and exponential forms with positive and negative exponents.

Learning Activities and Applications

- Using published advertisements for sales, students calculate prices including sales tax, percentage savings and actual savings (in dollars) for different items. They are given certain amounts of money to spend. (Basic)
- Students create board games in which moving forward a given number of spaces is represented by a positive integer and moving backward is represented by a negative integer. Students write rules for the game and use a spinner or number cube to indicate how many spaces a player moves. Students demonstrate their games to the class. (Proficient)
- Students use patterns to write numbers with negative exponents in standard form and write numbers in scientific notation. Then students research the uses of very small numbers and create and solve problems that require computation with the numbers written with negative exponents. (Advanced)

Proficiency Level Illustrations for Content Indicator 2

Basic

2-1 Christy has 88 photographs to put in her album. If 9 photographs will fit on each page, how many pages will she need? (NAEP-multiple choice)

2-2 [*Done without calculator*] The Quality Control Department of a company checks with customers and finds that 4,000 of the 200,000 buyers had a defective product. The percent of defective products was: (OCTM)

Ⓐ 0.02% Ⓑ 0.20% Ⓒ 2% Ⓓ 20%

2-3 Which of the following best approximates the number of hours in a month? (NAEP)

Ⓐ 350 Ⓑ 750 Ⓒ 1,000

Proficient

2-4 The lowest point of the St. Lawrence River is 294 feet below sea level. The top of Mt. Jacques Cartier is 1,277 feet above sea level. How many feet higher is the top of Mt. Jacques Cartier than the lowest point of the St. Lawrence River? Show your work. (NAEP)

2-5 On a winter day, the temperature is -2°C. As evening progresses, the temperature drops 5°C. Several hours later, the temperature rises by 3°C. What is the temperature after these changes? (OCTM)

Ⓐ -4°C Ⓑ -1°C Ⓒ 0°C Ⓓ 4°C

2-6 A calculator sells for $9.99 in a certain state. The purchase price including tax is $10.69. To the nearest whole number percent, which of the following is the best estimate of the sales tax in the state? (NAEP)

Ⓐ 7% Ⓑ 6% Ⓒ 5%

Advanced

2-7 Simplify $x^2y \div xy^2$ using negative exponents. (Panel)

2-8 Of the following, which is the closest to $2 + \sqrt{1{,}000}$? (Panel)

Ⓐ 32 Ⓑ 102 Ⓒ 502

2-9 Describe how patterns *a* and *b* are related. Then write the next 3 terms of pattern *b*. (Panel)

a	*b*
4	16
3	8
2	4
1	2
0	
-1	
-2	

3 Determine greatest common factor, least common multiple and prime factorization of numbers.

Mastering the concepts of greatest common factor and least common multiple prepares students for factoring and simplifying rational expressions and equations in algebra. Manipulating the terms in expressions and equations, as well as recognizing common factors, are basic algebraic skills. In the middle grades, students must master these skills. Also, they need to understand that whole numbers can be written as products of factors. This, along with the properties of operations, will help students make a successful transition to proportional reasoning and simplifying algebraic expressions and equations. Students should recognize cases in which simplifying fractions and numerical expressions can help make computation easier or unlock the solution to a problem.

Students who are having difficulty in determining the greatest common factor and the least common multiple when combining and simplifying fractions with unlike denominators may benefit from using fraction strips and/or arrays of counters (a given number of counters arranged in a square or a rectangle). Fraction circles may also help students understand renaming mixed numbers and whole numbers as improper fractions. Some students may need extra help and time outside of their regular mathematics classes to gain confidence in these concepts.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Find and use factors and multiples. ▫ Write powers with bases and exponents. ▫ Recognize and use the divisibility rules for 2, 5, and 10.	▫ Identify prime and composite numbers. ▫ Use factor trees to write the prime factorization of numbers with exponents. ▫ Use divisibility rules for 3 and 4.	▫ Use the divisibility rules for 6, 8, 9, and 12. ▫ Solve word problems involving greatest common factor, least common multiple, and prime factorization.

Learning Activities and Applications

- Students write numbers for others to identify as divisible by 2, 5, and/or 10. Students verify their answers by checking with a calculator and explain how they know whether the number is divisible by 2, 5, and/or 10 or not. (Basic)
- Students write the prime factorization of numbers using exponents. Then they explain any patterns especially with powers of a given base, if any, they find to the class. (Proficient)
- Students research the history of the search for methods to identify and factor large prime numbers, including current efforts that use computers. (Advanced)

Proficiency Level Illustrations for Content Indicator 3

Basic

3-1 Find the greatest common factor of 24 and 32. (Panel)

3-2 Which is the least common multiple of 8, 12, and 15? (Panel)

Ⓐ 60 Ⓑ 8 Ⓒ 15 Ⓓ 3 Ⓔ 120

Proficient

3-3 Write the prime factorization of 856 with exponents. (Panel)

3-4 For each of the numbers listed below, name at least one factor other than 1 or itself. (Panel)

Ⓐ 321 Ⓑ 264 Ⓒ 498

3-5 Which number is prime? (Panel)

Ⓐ 0 Ⓑ 1 Ⓒ 51 Ⓓ 53

Advanced

3-6 Two runners start running at the same time from the start/finish line of a 400-meter oval track. One runner runs laps of 1 minute 15 seconds and the other runs laps of 1 minute 45 seconds. How long will it be before the runners cross the start/finish line at the same time? How many laps will each have run? Explain your answer. (Panel)

3-7 A principal wants to send representatives of all the school's performing arts clubs and the journalism club to a Broadway musical on tour in a nearby city. Because she wants to be fair, she wants to make sure that the numbers of students from each group who attend are exactly proportional to the numbers of students in each group. Based on the number of students in each group listed below, and assuming that a student can belong to only one group, is this possible? Why or why not? (Panel)

Group	Number of students
Chorus	42
Dance Club	15
Drama Club	21
Journalism Club	12
Orchestra	47
Wind Ensemble	51

4 Write and use ratios, rates and proportions to describe situations and solve problems.

Mastering ratio and proportion is essential for making the transition from arithmetic to topics in algebra, geometry and beyond. Ratios and proportions provide the conceptual underpinnings for finding unknown measurements in similar figures, immeasurable distances, analyzing and representing functions, investigating slope, reductions and enlargements of figures, and numerous other applications. Ratio is fundamental to understanding whether or not an algebraic function is linear or not. In the middle grades, it is essential that students master proportional reasoning to prepare them for the thinking and problem solving they will need in their further study of mathematics.

To help students gain a basic understanding of ratio it is helpful to use familiar contexts. For example, altering the quantity of the ingredients in a recipe must be proportional in order to increase or decrease the number of servings without changing the taste or success of the dish. An enlargement of a photograph will have a different appearance if the length and width are not both increased proportionally. (The latter can be demonstrated easily using most graphic or photographic software applications.)

Students must have numerous experiences solving for missing values in proportions in addition to writing and solving proportions in a variety of problem-solving contexts. Again, using number frames (boxes or answer blanks used for unknown values) may be useful. Proficiency in proportional reasoning can be further developed by comparing similar geometric figures using measurement tools and by graphing functions, such as distance, rate and time. Graph paper, graphing calculators and graphing software can aid students' understanding of ratio and proportion.

Benchmark Proficiency Progression

Basic

- Write ratios, rates and proportions based on simple given situations.
- Solve for missing values in proportions.

Proficient

- Solve for missing measures in similar figures.
- Use ratios to determine whether geometric figures are or are not similar.
- Use ratios and proportions to find measures indirectly.
- Use scale factors to make scale drawings and maps.

Advanced

- Write and solve proportions in word problems and in geometric applications.
- Use right triangle trigonometric ratios for sine, cosine and tangent.
- Analyze scale drawings and maps for accuracy.

Learning Activities and Applications

- Using written recipes, students write new ingredient lists (and possibly pan sizes) based on a change (both increase and decrease) in a quantity of one of the ingredients (e.g., if they had only 2 tomatoes instead of the 3 called for in the recipe) and/or in the number of servings. (Basic)
- Students create spreadsheets and graphs comparing the population of states to the number of their representatives in the House of Representatives and the Senate. Which states appear to be the most and least powerful in the House? In the Senate? What is the historical justification for the difference in proportionality? (Proficient)
- Using the standings in a sports league at different points throughout the season, students project the won-lost records of teams at the end of the season based on their records at those points. At the end of the season, they review their different projections, compare them to the final records and create related graphs. (Proficient)
- Using scientific or graphing calculators, students complete tables for varying values of A as follows.

angle A	sin A	cos A	$\frac{\sin A}{\cos A}$	tan A

Then students write a conclusion based on what they observed about the values in their tables. (Advanced)

Proficiency Level Illustrations for Content Indicator 4

Basic

4-1 If $\frac{2}{25} = \frac{n}{500}$, then $n =$ (NAEP-multiple choice)

4-2 In a group of 1,200 adults, there are 300 vegetarians. What is the ratio of nonvegetarians to vegetarians in the group? (NAEP)

(A) 1 to 3 (B) 1 to 4 (C) 3 to 1 (D) 4 to 1 (E) 4 to 3

4-3 If $\frac{10.3}{5.62} = \frac{n}{4.78}$, then, of the following, which is closest to n? (Calculator use permitted) (NAEP)

(A) 2.61 (B) 3.83 (C) 8.76 (D) 8.82 (E) 12.11

Proficient

4-4 Luis mixed 6 ounces of cherry syrup with 53 ounces of water to make a cherry-flavored drink. Martin mixed 5 ounces of the same cherry syrup with 42 ounces of water. Who made the drink with the stronger cherry flavor? Give mathematical evidence to justify your answer. (NAEP)

4-5 In the model town that a class is building, a car 15 feet long is represented by a scale model 3 inches long. If the same scale is used, a house 35 feet high would be represented by a scale model how many inches high? (NAEP)

(A) $\frac{45}{35}$ (B) 3 (C) 5 (D) 7 (E) $\frac{35}{3}$

Advanced

4-6 A certain machine produces 300 nails per minute. At this rate, how long will it take the machine to produce enough nails to fill 5 boxes of nails if each box will contain 250 nails? (NAEP)

(A) 4 min (B) 4 min 6 sec (C) 4 min 10 sec (D) 4 min 50 sec (E) 5 min

4-7 A tennis court is 27 feet wide (for singles) and 78 feet long and the net is 3 feet high at the center of the court. A table tennis table is 5 feet wide and 9 feet long and the net is 6 inches high. Are a table tennis table and net proportional to a tennis court and net? Explain your answer.

If you wanted to make a true miniature version of a tennis court, if you used the width of the table tennis table (5 feet), how long would it be? How high would the net be? Finally, if you were to make a proportional model of yourself for your miniature tennis court, how tall would it be? (Panel)

5 Draw with appropriate tools and classify different types of geometric figures using their properties.

In the middle grades, students' geometric knowledge and skills should expand and become more formalized. Students need to move from learning about geometric shapes and objects to learning how to classify them based on their distinguishing characteristics. Making the connection between algebra and geometry is essential for further study in mathematics because algebraic solutions and their related geometric solutions to problems are essential for building the bridge from algebra and geometry to calculus. As students gain understanding of different types of angles and lines and their transversals, they can solve problems involving missing angle measures. Requiring students to think about and describe their reasoning will build the conceptual basis for understanding the formal structure of algebraic and geometric proof. Before students leave the middle grades, they should begin to construct basic logical arguments and use deductive and inductive reasoning.

Instruction in the middle grades should expand students' understanding of the properties of both regular and irregular plane figures and culminate with students being able to perform simple transformations of figures in the coordinate plane. Students' experiences with classifying basic geometric figures should be expanded to include investigations into their properties. Students should master drawing and measuring angles with a protractor, measuring length with a ruler, and performing one-step transformations of geometric figures. Using pattern blocks, graph paper and graphing software can help students understand the properties of figures and their transformations.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Identify, draw and classify geometric plane and solid figures. ▫ Perform transformations of figures. ▫ Use a protractor to draw and measure. ▫ Identify different types of angles and triangles.	▫ Recognize and write valid statements using "if-then," "all," "some," and "none" about geometric figures. ▫ Classify geometric figures using their properties. ▫ Predict the outcomes of composite transformations.	▫ Use knowledge of properties of geometric figures to construct and explain basic deductive arguments. ▫ Determine measures of angles formed by parallel lines and transversals. ▫ Perform transformations in the coordinate plane.

Learning Activities and Applications

- Students identify and classify geometric figures found in everyday objects (e.g., city streets and blocks, sports fields, windows in their classrooms, decorative patterns). (Basic)
- Students write statements using *all*, *some* and *none* about geometric figures, such as:

  All right triangles are scalene triangles.

  Some right triangles are scalene triangles.

  No right triangles are scalene triangles.

  Then they determine which statements are true and which are false. Students share their statements with the class. (Proficient)
- Using graph paper and tagboard, students make models of various geometric plane figures such as rectangles and isosceles triangles. They experiment with translating the figures in the coordinate plane (on graph paper) and record all of their observations systematically in tables by indicating the coordinates of each figure at the starting position and the ending position. They observe any patterns and formulate conclusions about the changes in the coordinates. They share their observations with the class. (Advanced)

Proficiency Level Illustrations for Content Indicator 5

Basic

5-1 Which is not a property of a square? (Panel)

(A) All sides are congruent. (B) All angles are congruent.

(C) Opposite angles are congruent. (D) Opposite angles are complementary.

5-2 Using a ruler and a protractor, draw a right triangle. (Panel)

5-3 Make a drawing showing when a slide of a figure is the same as a flip. (Panel)

Proficient

5-4 Explain how you could use each of the sets of tools listed in the table to determine whether figures you are given are squares, rectangles, right triangles, or parallel lines. (Panel)

Figure	Ruler	Protractor, unmarked straightedge, and pencil
Rectangle		
Square		
Right triangle		
Parallel lines		

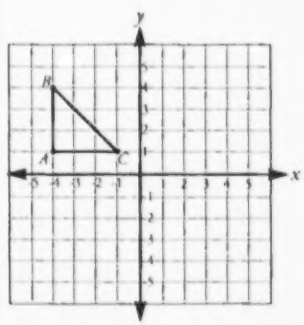
5-5 Which is not the ratio of the length of a side of an equilateral triangle to its perimeter? (OCTM-free response)

(A) 1 : 3 (B) 3 to 1 (C) $\frac{1}{3}$ (D) 1 to 3

Advanced

5-6 The two angles of a triangle measure 45° each. Can the triangle be a scalene triangle? Explain why or why not. Can the triangle be an equilateral triangle? Explain why or why not. Use the number of degrees in the sum of the measures of a triangle to classify the triangle by the measure of its angles and the lengths of its sides. (Panel)

5-7 Triangle ABC is rotated 90° clockwise about the origin. What are the new coordinates of B? (Panel)



(A) (-5, 4) (B) (5, 4) (C) (4, 5) (D) (-4, 5)

6 Measure length with appropriate tools and find perimeter, area, surface area and volume using appropriate units, techniques, formulas and levels of accuracy.

Middle grades instruction should focus on solidifying students' measurement skills, including knowledge and application of appropriate units and levels of accuracy. Students' mastery of basic geometric formulas is related to other topics in mathematics, including ratio and proportion, identifying figures and angles, writing and solving algebraic equations, and understanding functions. Students need numerous experiences with word problems throughout the middle grades, particularly those dealing with real-world situations involving formulas. Prior to their study of Algebra I, students should begin to understand that formulas are equations. While middle grades students need to develop the ability to apply formulas, students also need to commit a certain number of facts and formulas to memory, such as the fact that π is approximately equal to 3.14, and that the sum of the measures of the three angles of a triangle is 180 degrees.

Some students may need extra help and time to gain facility in using a ruler, metric and customary, and the formulas for the perimeter, area, and volume of basic geometric figures. While using a ruler is a skill that should be mastered in the elementary grades, many middle grades mathematics teachers report that their students have difficulty measuring length. Although higher level mathematics tends not to involve taking direct measurements, the inability to use a ruler may signify conceptual weaknesses. These weaknesses could be difficulty with number lines and/or one-to-one correspondence or not understanding the concept of measurement as the process of assigning a number to a characteristic of an object or figure. These two concepts are very important in the further study of mathematics. Using various hands-on materials such as graph paper, centimeter cubes, geoboards and graphing software can help students understand the basic concepts of distance, perimeter, area and volume.

Benchmark Proficiency Progression

Basic

- Recognize which units of measure are most appropriate for a given situation.
- Measure and calculate the perimeter and area of regular polygons.
- Calculate the volume and surface area of cubes and rectangular solids.
- Calculate the circumference and area of circles.
- Read and use rulers, protractors and compasses.

Proficient

- Calculate the surface area and volume of cylinders.
- Find perimeter and area of combined figures.
- Solve problems involving capacity and volume (e.g., number of books of the same size that fit in a given box).

Advanced

- Calculate the volume of spheres and cones.
- Apply a level of accuracy using significant digits.

Learning Activities and Applications

- Students develop a plan to estimate the cost of painting the inside and outside walls of their school with two coats of paint (using brushes and rollers), using measurements from their classroom *only* and the exterior of their school. They can research or be given the cost of paint and the area covered by a can of paint. They can also develop a similar plan to estimate the time it would take to complete the job. (Basic)
- Students use graph paper to find all of the possible nets of a cube, rectangular solid and triangular pyramid. They share their results with the class. (Proficient)
- Students use the formula [density = mass ÷ volume] to create problems involving the mass, density and volume of various solids constructed from metals such as gold, silver, iron and so on. Students research the density of various metals and use the relationship between mass and volume in the metric system. For example, a jeweler is making a gold sphere with a radius of 2 centimeters. The cost of the sphere is determined by the weight (mass) of the gold the jeweler uses. What is the mass of the gold sphere? (Advanced)

Proficiency Level Illustrations for Content Indicator 6

Basic

6-1 A rectangular room is 8 feet high, 10 feet wide, and 12 feet long. Emile buys wallpaper for the walls. Disregarding doors, windows, and pattern matching, what is the minimum area of wallpaper he could use to cover all of the walls? (OCTM)

Ⓐ 352 square feet Ⓑ 400 square feet Ⓒ 592 square feet Ⓓ 960 square feet

6-2 A rectangular carpet is 9 feet long and 6 feet wide. What is the area of the carpet in square feet?(NAEP)

Ⓐ 15 Ⓑ 27 Ⓒ 30 Ⓓ 54

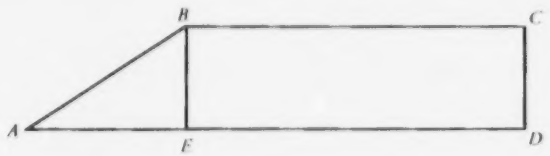
6-3 Of the following, which is the best unit to use when measuring the growth of a plant every other day during a 2-week period? (NAEP)

Ⓐ centimeter Ⓑ meter Ⓒ kilometer Ⓓ foot Ⓔ yard

Proficient

6-4 A cube has sides that measure 8 inches. If the length of one of the sides is increased by 25 percent, what is the percentage increase in the volume? (Panel)

6-5 The perimeter of rectangle *BCDE* is 32 inches. If the length of the rectangle is 3 times its width and area of triangle *ABE* is 6 square inches, what is the perimeter of trapezoid *ABCD*? (Panel)



6-6 Plastic edging for flower beds comes in 50-foot rolls and costs $6.85 per roll. What is the cost to completely edge two rectangular flower beds 40 feet by 15 feet and one circular flower bed 16 ft in diameter? (OCTM)

Ⓐ $13.70 Ⓑ $27.40 Ⓒ $34.25 Ⓓ $41.10

Advanced

6-7 It takes 64 identical cubes to **half** fill a rectangular box. If each cube has a volume of 8 cubic centimeters, what is the volume of the box in cubic centimeters? (NAEP)

Ⓐ 1,024 Ⓑ 512 Ⓒ 128 Ⓓ 16 Ⓔ 8

6-8 A technician at a well-baby clinic measures and records the lengths of infants at their regular check-ups during their first year. Describe, using if-then statements, how accurate the measurements must be to record a baby's growth. (Panel)

7 Understand and use the Pythagorean relationship to solve problems.

The Pythagorean relationship has widespread applications in science and in technical and engineering fields. The Pythagorean relationship is used throughout the study of mathematics, especially in algebra, geometry and trigonometry. Finding immeasurable distances is a typical application of the Pythagorean relationship.

It is helpful if students have already learned the properties of right triangles and have the ability to find squares and square roots. The difficulty could come as students encounter applications in which it is not obvious that the Pythagorean relationship should be used. Encouraging students to make diagrams of the facts in problems helps them visualize the existence of right triangles and thus leads them to use the Pythagorean relationship to find missing distances or lengths.

Because computing with squares and square roots is something more easily done on a calculator, Pythagorean-relationship problems provide a valuable opportunity for students to improve their calculator skills. At the same time, students should understand that calculators are not always necessary, especially if the problem involves perfect squares or expressing an answer in radical form. In place of using calculators, students can learn to estimate square roots by averaging.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Memorize the Pythagorean relationship. ▫ Recognize when to use the relationship. ▫ Find the measure of the third side in right triangles given the measures of two sides.	▫ Solve word problems involving right triangle measures. ▫ Use the Pythagorean relationship to determine whether triangles are right triangles. ▫ Memorize Pythagorean triples to solve problems without a calculator.	▫ Solve problems where the existence of triangles is not obvious. ▫ Find unknown distances across lakes, etc. using the relationship.

Learning Activities and Applications

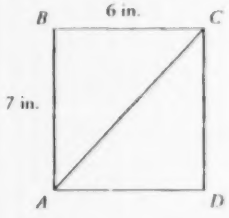
- Using graph paper, students outline large squares on each side of a right triangle and then cut out the small squares along the legs to see if the small squares fit inside the large square along the hypotenuse. (Basic)
- Students choose several tall landmarks in the local area (buildings, trees, etc.) and estimate their heights, measure them using the methods and instruments they researched, and compare their estimates and measurements. (Proficient)
- Students research the various proofs of the Pythagorean relationship and make displays showing the methods they found. Students demonstrate their findings to the class. (Advanced)

Proficiency Level Illustrations for Content Indicator 7

Basic

7-1 A carpenter claims that the wooden deck just completed is a square. The home owner measured the sides as 30 feet and 40 feet, and the diagonal as 50 feet. Is the deck a square? Explain whether or not you would use the Pythagorean relationship to answer the question. (Panel)

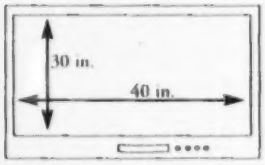
7-2 Of the following, which is the closest approximation of the length of *AC* in the rectangle below? (NAEP)



(A) 7 inches (B) 9 inches (C) 13 inches

Proficient

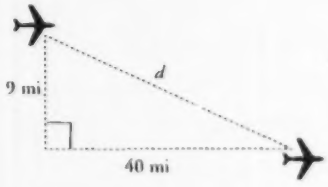
7-3 What is the diagonal measurement of the TV screen shown in the figure below? (NAEP)



(A) 25 in. (B) 35 in. (C) 50 in. (D) 70 in. (E) 1,200 in.

7-4 Is a triangle with sides measuring 3 inches, 5 inches, and 7 inches a right triangle? Explain your answer. (Panel)

7-5 How far apart are the planes? (Panel)

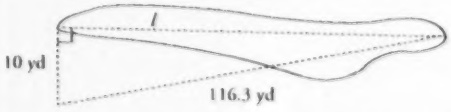


(A) 81 miles (B) 1,600 miles (C) 1,681 miles (D) 41 miles

Advanced

7-6 Two students are using a measuring tape to measure the length of a room. They measure it to be 13 feet $6\frac{1}{4}$ inches. However, the student at one end is holding the tape 9 inches higher than the student at the other end. What is the difference between their measurement and the true length? (Panel)

7-7 What is the approximate length of the pond? (Panel)



(A) 2,500 yards (B) 11,025 yards (C) 116 yards (D) 13,525 yards

8 Gather, organize, display and interpret data.

Besides basic computation, data analysis is arguably the area of mathematics people are most likely to encounter in their daily lives. Usually people do not collect their own data, but they are often faced with graphs, charts, survey results and statements regarding statistical information in newspapers, magazines, statements of political candidates and government officials, and report cards and standardized test data. In addition, data analysis is useful in other content areas, especially science and social studies. Data analysis becomes more sophisticated when students study algebra. Here they learn that most relationships between two variables are not just exactly linear but approximate linear functions. Students also learn that sometimes data relationships are non-linear.

In the middle grades, data analysis provides an ideal context for applying, assessing and reinforcing other mathematics skills and concepts. Data analysis should be made more challenging by integrating numerous other mathematics topics and creating the foundation for more advanced work in high school. Middle grades students should have experiences calculating percent change, using geometric concepts to make circle graphs, communicating mathematically by presenting their interpretations of data, using logical reasoning to analyze statistical claims, and plotting points to create line graphs. Using technology such as graphing software and calculators can help students display and interpret data.

Some students may need extra time and help to improve their data interpretation skills. Using smaller numbers, fewer data items in a set and guiding students to choose the appropriate display for the types of data sets are some ways to help them. Encouraging students to use organizing displays such as stem-and-leaf plots and line plots can help them find measures of central tendency and range.

Benchmark Proficiency Progression

Basic

- Make and read single bar graphs, single line graphs, and pictographs.
- Read and interpret circle graphs.
- Find the mean, median, mode, and range of sets of data.
- Plot points on a coordinate grid.

Proficient

- Read and make line plots and stem-and-leaf plots.
- Collect and display data for given situations.
- Make, read and interpret double bar, double line, and circle graphs.
- Determine when to use mean, median, mode, or range.
- Determine and explain situations of misleading statistics.

Advanced

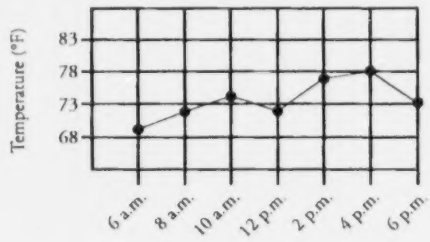
- Formulate survey questions and collect data.
- Evaluate statistical claims in articles and advertising.
- Analyze different displays of the same data.

Learning Activities and Applications

- Students research the Dow Jones Industrial Average. What does it measure? What is it an average of? What are the units of measure? Students explain whether or not one can predict future performance from a graph of it. (Basic)
- Students compare graphs appearing in newspaper and magazine articles and those in advertisements. How are they different? Is one type clearer than the other? How are the graphs used in the articles and in the advertisements? Do the graphs in advertisements lead the reader to certain conclusions? Do the graphs from newspaper articles? Students identify and describe the conclusions, if any. (Proficient)
- From the same data sets, students create appropriate graphs of different types. They analyze the different graphs and compare and contrast the conclusions from each. (Advanced)

Proficiency Level Illustrations for Content Indicator 8

8-1 According to the graph below, the temperature at 10 a.m. is approximately how many degrees greater than the temperature at 8 a.m.? (NAEP)



(A) 1 (B) 1.5 (C) 2 (D) 2.5 (E) 3

8-2 The graph below shows the number of traffic tickets that were issued in City *X* each month of 1988. Approximately how many were issued during the entire year? (NAEP)

(A) 800 (B) 1,200 (C) 1,600

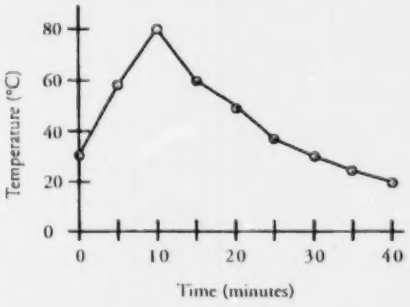
Basic

BEST COPY AVAILABLE

Proficiency Level Illustrations for Content Indicator 8 Continued ...

Proficient

8-3 The graph below best conveys information about which of the following situations over a 40-minute period of time? (NAEP)



(A) Oven temperature while a cake is being baked

(B) Temperature of water that is heated on a stove, then removed and allowed to cool

(C) Ocean temperature in February along the coast of Maine

(D) Body temperature of a person with a cold

(E) Temperature on a July day in Chicago

8-4 Akira read from a book on Monday, Tuesday, and Wednesday. He read an average of exactly 10 pages per day. Indicate in the boxes below whether each of the following is possible or not possible. (NAEP)

Pages Read				
Monday	Tuesday	Wednesday	Possible	Not Possible
4 pages	4 pages	2 pages	☐	☐
9 pages	10 pages	11 pages	☐	☐
5 pages	10 pages	15 pages	☐	☐
10 pages	15 pages	20 pages	☐	☐

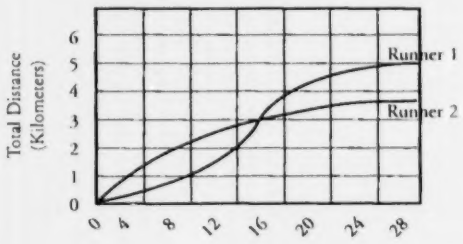
8-5 The yearly salaries of the five top executives at the Bigwig Corporation are $1,000,000; $250,000; $130,000; $90,000; and $90,000. If we calculate the mean, median, and mode for these salaries and then place these values in order from highest to lowest, the order would be: (OCTM)

(A) mean, median, mode (B) mode, median, mean

(C) median, mean, mode (D) mean, mode, median

Proficiency Level Illustrations for Content Indicator 8 Continued ...

8-6 The total distances covered by two runners during the first 28 minutes of a race are shown in the graph below. How long after the start of the race did one runner pass the other? (NAEP)



(A) 3 minutes
(B) 8 minutes
(C) 12 minutes
(D) 14 minutes
(E) 28 minutes

8-7 In 1980, the populations of Town *A* and Town *B* were 5,000 and 6,000, respectively. The 1990 populations of Town *A* and Town *B* were 8,000 and 9,000, respectively.

Brian claims that from 1980 to 1990 the populations of the two towns grew by the same amount. Use mathematics to explain how Brian might have justified his claim.

Darlene claims that from 1980 to 1990 the population of Town *A* had grown more. Use mathematics to explain how Darlene might have justified her claim. (NAEP)

1980 Population	1990 Population
Town *A* ☺☺☺☺☺	Town *A* ☺☺☺☺☺☺☺☺
Town *B* ☺☺☺☺☺☺	Town *B* ☺☺☺☺☺☺☺☺☺

☺ = 1,000 people

8-8 The table below provides information about the cost of placing phone calls between certain cities at different times during the day. How much more would it cost to place a 10-minute call from Allenville to Edgeton at 3 p.m. on Friday than at 3 p.m. on Saturday? (NAEP)

Telephone Calling Rates

	Day Rate 7 a.m. - 5 p.m. Mon-Fri		Evening Rate 5 p.m. - 11 p.m. Mon-Fri 8 a.m. - 11 p.m. Sat-Sun		Night Rate 11 p.m. - 8 a.m. ALL DAYS	
From Allenville to	First Minute	Each Additional Minute	First Minute	Each Additional Minute	First Minute	Each Additional Minute
Burneyford	$.09	$.03	$.07	$.02	$.05	$.02
Camptown	$.28	$.09	$.22	$.07	$.17	$.05
Dorning	$.37	$.11	$.30	$.09	$.22	$.07
Edgeton	$.42	$.12	$.34	$.10	$.25	$.07

Advanced

9 Determine the number of ways an event can occur and the associated probabilities.

Usually students enter the middle grades with some understanding of the probability of simple events. However, students often think that an independent event can be influenced by preceding trials, for example, that a coin that has landed on heads several times in a row is either more likely to land on heads, because that is what has been happening in previous trials, or more likely to land on tails because it is "due" to do so. Or they may not apply their knowledge of probabilities to situations and instead rely on subjective judgments. Students' understanding of the basic concepts involved in probability lays the groundwork for further study in algebra of combinations, permutations, prediction and fair games.

To help them understand the basic concepts of probability, students should have numerous hands-on experiences where they predict outcomes, conduct experiments, construct sample spaces, record and compare their results to mathematical probabilities. Two-color counters, spinners, number cubes and other such materials are useful in constructing experiments. It is very important to emphasize the number of trials, fairness of the materials used and keeping a record of the outcomes.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Find the probability of simple events. ▫ Find the sample space using tree diagrams. ▫ Find the number of possible outcomes.	▫ Find and record the experimental probability of multiple trials of simple events. ▫ Identify and make predictions using theoretical probabilities. ▫ Find combinations.	▫ Identify the sample space and mathematical probability for compound events and permutations. ▫ Distinguish between independent and dependent events. ▫ Recognize fair and unfair games.

Learning Activities and Applications

- Students predict the outcomes and then toss a 1 to 6 number cube 100 times and record their results. Then they compare their results with the mathematical probability of tossing a 1 to 6 number cube. They share their results with the class. The activity could be extended by having students predict and then toss a 1 to 4 tetrahedron. (Basic)
- Students conduct different experiments for the same event but with different numbers of trials. For example, one group tosses a two-color counter 10 times, another group 20 times, another group 30 times, and so on. They compare the different experimental probabilities, represent them graphically and write a conclusion. (Proficient)
- Students calculate the total possible number of telephone numbers in their area code. In one activity, they devise a plan to estimate the number of working numbers a telemarketer could expect to reach by placing 100 calls at random. In another activity, they devise a plan to investigate whether their area will "run out" of numbers in the near future and propose and evaluate different solutions (e.g., adding another digit, using the # and * keys). (Advanced)

Proficiency Level Illustrations for Content Indicator 9

Basic

9-1 The nine chips shown below are placed in a sack and then mixed up. Madeline draws one chip from this sack. What is the probability that Madeline draws a chip with an even number? (NAEP)

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{4}{9}$ (D) $\frac{1}{2}$

9-2 How many possible outcomes are there when choosing a letter in the word STATES? What is the number of favorable outcomes for the event of choosing the letter S? (Panel)

Proficient

9-3 A coin is to be tossed 3 times. What is the probability that 2 heads and 1 tail in any order will come up? (NAEP)

9-4 Dave will choose one sandwich and one drink for lunch. The menu at left shows the choices. List below all the possible combinations of a sandwich and a drink that Dave might choose. (NAEP)

MENU

Sandwiches	Drinks
Beef	Milk
Tuna	Juice
Ham	

9-5 A bag contains 100 marbles. Some marbles are red and some are yellow. Suppose a student without looking chooses a marble out of the bag, records the color, and then places the marble back in the bag. The student has recorded 8 red marbles and 17 yellow. Using these results which is the best prediction of how many red marbles are in the bag? (Panel)

(A) 25 (B) 11 (C) 32 (D) 68

Advanced

9-6 The two spinners shown below are part of a carnival game. A player wins a prize only when both arrows land on black after each spinner has been spun once. James thinks he has a 50-50 chance of winning. Do you agree? Justify your answer. (NAEP)

9-7 Of the 60 people in a room, $\frac{2}{3}$ are men and $\frac{3}{5}$ of the people have brown hair. What is the least number of men in the room who could have brown hair? Solve this problem in two ways. In one way, use a drawing in your solution. Explain both ways. (OCTM)

10 Write, simplify and solve algebraic equations using substitution, the order of operations, the properties of operations and the properties of equality.

Experiences that serve as building blocks for algebra are, for example, finding missing values in number frames (e.g., 4 x □ = 32), using formulas, the properties of operations, the order of operations, and graphing ordered pairs. During the middle grades, the emphasis should focus on students using algebraic symbols to write and solve simple equations in one variable in problem-solving contexts. They should learn the properties of equality and the order of operations to simplify and solve equations. Also, students should have some experiences in solving equations and inequalities in two variables and a basic introduction to the concept of functions. It is key that students understand that the graph of an equation is its "picture" so that they begin to understand the relationship between algebra and geometry for their further studies in mathematics.

Students may benefit from using counters, algebra tiles, and equation mats to help them solve simple equations in one variable. For simple equations and inequalities in two variables, students should use tables to record their computations for solution pairs and graph paper for their graphs. Graphing calculators and software can help students understand the concepts of linear functions and relations.

Benchmark Proficiency Progression

Basic

- Explore the concept of and different uses of variables.
- Solve one- and two-step equations and inequalities in one variable.
- Write and solve equations in one variable for word problems.
- Know and apply the properties of operations.

Proficient

- Graph equations in two variables using graphing calculators.
- Solve equations in two variables algebraically.
- Understand and use the properties of equalities and inequalities.

Advanced

- Graph linear inequalities using graphing calculators.
- Write and solve problems using systems of linear equations.
- Graph systems of linear equations to find their solutions using graphing calculators.
- Graph linear equations using the slope-intercept method ($y = mx + b$) using graphing calculators.

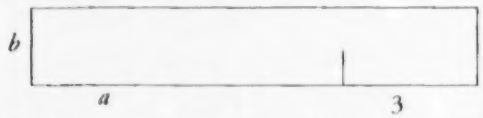
Learning Activities and Applications

- Students write as many equivalent equations as they can for a given equation such as $x + 7 = 10$ or $15 = 3 + y$. They use algebra tiles and equation mats to verify their answers and then solve each equation. (Basic)
- Using graphing calculators, students investigate the slopes of several different linear equations to determine how the graphs change when the sign and value (including the reciprocal) of each slope changes. Students record their results and present their findings to the class. (Proficient)
- Students examine different offers from CD clubs, Internet providers or mobile telephone services and represent them using algebraic equations and graphs. They then look at different scenarios (e.g., different numbers of CDs bought per month, different amounts of time spent on the Internet or using the phone each month) and make statements about when each appears to be cost-effective and ineffective. Looking at two different offers of the same type (e.g., two different CD clubs), they try to identify points where the costs are equal and where one is better than the other. (Advanced)

Proficiency Level Illustrations for Content Indicator 10

Basic

10-1 What expression can you write for the perimeter of the rectangle below? (OCTM)



10-2 Write two numbers that could be put in the ☐ to make 54 < 3 x ☐ true. (NAEP)

10-3 If $n + n + n = 60$, what is the value of n? (NAEP)

10-4 If ☐ represents the number of newspapers that Lee delivers each day, which of the following represents the total number of newspapers that Lee delivers in 5 days? (NAEP)

Ⓐ 5 + ☐ Ⓑ 5 x ☐ Ⓒ ☐ ÷ 5 Ⓓ (☐+☐) x 5

10-5 Given 3•(☐ + 5) = 30, the number in the box should be: (OCTM)

Ⓐ 2 Ⓑ 5 Ⓒ 10 Ⓓ 95

Proficient

10-6 Graph the following equation: $3y = 6x + 12$ (Panel)

10-7 The cost to rent a motorbike is given by the formula: Cost = ($3 x number of hours) + $2 (NAEP)

Fill in the table at right.

Time in Hours	Cost in Dollars
1	5
4	
	17

10-8 Find x if $10x - 15 = 5x + 20$ (Panel)

10-9 A plumber charges customers $48 for each hour worked plus an additional $9 for travel. If h represents the number of hours worked, which of the following expressions could be used to calculate the plumber's total charge in dollars? (NAEP)

Ⓐ $48 + 9 + h$ Ⓑ $48 \times 9 \times h$ Ⓒ $48 + (9 \times h)$ Ⓓ $(48 \times 9) + h$ Ⓔ $(48 \times h) + 9$

Advanced

10-10 Graph the following inequality: $7y + 14 < 6x + 16$ (Panel)

10-11 If $d = 110$ and $a = 20$ in the formula $d = \frac{a}{2}(2t - 1)$, then $t =$ (NAEP)

Ⓐ $\frac{15}{2}$ Ⓑ $\frac{15}{18}$ Ⓒ 5 Ⓓ $\frac{111}{20}$ Ⓔ 6

11 Represent, analyze, extend and generalize a variety of patterns.

The study of patterns in the middle grades should progress from patterns with pictures or simple number sequences to patterns that relate to linear functions. Linear functions indicate constant rates of change and are basic to the study of algebra and geometry. Students also should move from informal descriptions of patterns to writing algebraic rules for patterns. Students' experiences with using patterns to find the results when multiplying and dividing whole numbers and decimals by powers of 10 and demonstrating the laws of signed numbers will help them learn how to use algebraic symbols for patterns.

While finding the rule underlying a pattern may seem a matter of intuition, students can greatly increase their success with understanding patterns through extensive experiences. Using tables to systematically record their results when extending patterns will help students write rules for patterns. Practice with words that indicate increase and decrease will help students symbolize their understanding of particular patterns that show a constant rate of change.

Benchmark Proficiency Progression

Basic

- Complete and extend patterns with pictures or simple numerical progressions.
- Describe pattern rules informally.

Proficient

- Find missing values in patterns.
- Find values from rules of patterns.
- Analyze and create rules from patterns.
- Write pattern rules as algebraic statements.

Advanced

- Analyze and extend complex patterns (e.g., ones involving multiple operations and powers).
- Analyze and use patterns in other contexts (e. g., Pascal's triangle).

Learning Activities and Applications

- Provide students with a variety of number sequences that are arithmetic (the difference between two consecutive terms is the same), geometric (the terms are formed by multiplying by a constant factor), or neither. Have students explain the pattern informally and then extend it to the next three terms. (Basic)
- Provide students with two or three algebraic rules for patterns, such as, $2n - 1$ where n = the natural numbers, and have them write the first five terms and another term, such as the tenth term. Then have students write their own rules for sequences and verify their rules by finding terms of the sequence. (Proficient)
- Have students research Pascal's Triangle to find out what it is and how it is used. Have students describe the pattern of the triangle itself, the pattern of the sum of each of the rows, how it is used to find the probability of an event with two equally likely outcomes, and how it is used in finding combinations. (Advanced)

Proficiency Level Illustrations for Content Indicator 11

Basic

11-1 In the pattern, which figure would be next? ⊔ ⊏ ⊓ ⊐ ⊔ ⊏ ⊓ ⊐ ⊔ ? (NAEP)

(A) ⊔ (B) ⊏ (C) ⊓ (D) ⊐

11-2 Describe the rule for this pattern. 7, 4.5, 2, -0.5, -3, … (Panel)

Proficient

11-3 If the pattern shown in the table were continued, what number would appear in the box at the bottom of column *y* next to 15? (NAEP)

x	*y*
1	1
3	5
5	9
7	13
15	?

Answer: _____

11-4 A computer costing $1,500 loses one-third of its value each year. What is its value at the end of the third year? (Panel)

11-5 Which is the rule for this pattern? 1, 3, 9, 27, 81, … (Panel)

(A) $3n$ (B) 3^n (C) $3 + n$ (D) $n - 3$

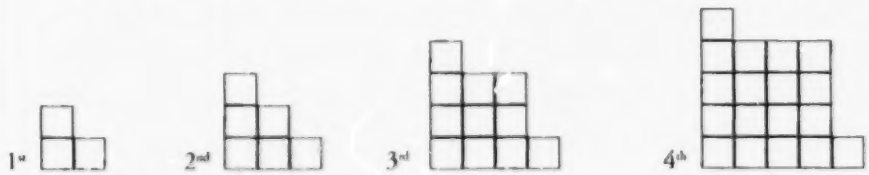
Advanced

11-6 Write a formula to find *y* for any value of *x* for the pattern shown in the table. (NAEP)

x	*y*
1	1
3	5
5	9
7	13
15	?

Answer: _____

11-7 If you continue the pattern below, how many blocks will be needed to construct the 20th figure in the sequence? Explain how you found your answer. (OCTM-multiple choice)



12 Understand and represent functions algebraically and graphically.

Functions represent one of the main bridges between middle grades mathematics and Algebra I. Functions integrate many other mathematics skills and concepts, such as formulas, coordinate graphing, patterns, rates and variables. The nature of change expressed by functions is better understood when students are exposed to a variety of different representations, such as graphs, verbal descriptions, equations and function tables. Graphing functions is particularly important in helping students appreciate the changes in values represented by functions and in helping them understand the formal definition of the relationship between the variables in a function. Graphing familiar formulas, such as the perimeter of a square or the volume of a cube, will help students understand the difference between linear and non-linear functions.

Using graph paper, graphing software and calculators can help students understand the concepts of functions and facilitate their understanding of the graphs and equations for linear and non-linear functions. Providing simple problem-solving contexts is also helpful.

Benchmark Proficiency Progression

Basic	Proficient	Advanced
▫ Graph functions from tables and patterns. ▫ Create tables and graphs of linear functions. ▫ Determine the output given the input.	▫ Explore functions using familiar formulas. ▫ Explore rates of change. ▫ Explore non-linear functions.	▫ Understand the definitions of functions and relations. ▫ Use the vertical line test. ▫ Explore inverse functions.

Learning Activities and Applications

- Students draw simple input/output diagrams and find values for the integers from -5 to 5. Students share their diagrams and results with the class. (Basic)
- Students research prices, fuel efficiencies and average annual maintenance costs of different cars. Using spreadsheet software, and assuming they only drive to and from school, they calculate the average annual operating expenses using a range of gas prices. They then create graphs to illustrate the differences. Are there some inexpensive cars that are relatively expensive to operate? Are there expensive cars that are relatively inexpensive to operate? (Proficient)
- The workers at a factory are on strike. Management estimates that the strike is a loss of $500,000 a day in business. Using these facts, have students predict scenarios in which the business must close, the impact of when workers return to work, and how long the workers can stay out on strike without the business closing its doors. Have students support their predictions with graphs and tables. (Advanced)

Proficiency Level Illustrations for Content Indicator 12

Basic

12-1 This table shows input/output values. According to the pattern, which is the input for an output of 20? (Panel)

Input	Output
4	2
6	3
8	4
12	6
16	8
?	20

(A) 40 (B) 10 (C) 100 (D) 5

12-2 For the equation $y = 3x + 8$, create a table of values for x and y. Use five different values of x to find the corresponding values of y. Draw a graph from your table. (Panel)

Proficient

12-3 This table shows the number of times a person blinks on average. Describe the rate of change in words. (Panel)

number of blinks	number of minutes
25	1
50	2
75	3
100	4
125	5

12-4 Which function is non-linear? (Panel)

(A) $y = 2x$ (B) $y = 2^x$ (C) $y = 2 + x$ (D) $y = \frac{x}{2}$

Advanced

12-5 One plan for a state income tax requires those persons with income of $10,000 or less to pay no tax and those persons with incomes greater than $10,000 to pay a tax of 6% only on the part of their income that exceeds $10,000. A person's effective tax rate is defined as the percent of total income that is paid in tax. Based on this definition, could any person's effective tax rate be 5%? Could it be 6%? Explain your answer. (Panel)

12-6 A new band, The Old Spice Boyz, has just made a CD. They made it themselves and it cost $12,000 to make. They have been offered contracts by two different record companies to produce and promote it. One company offers $12,000 and 16 percent of total sales. Another offers $50,000 but only 10 percent of total sales. At what amount of sales will the contracts give the band the same amount of income? (Panel)

Answers to Proficiency Level Illustrations

1-1 D

1-2 C

1-3 between D and E

1-4 C

1-5 2.3456×10^{10}

1-6 26, 27, 28, 29, 30, 31, 32, 33, 34, 35

1-7 A

2-1 10 pages

2-2 C

2-3 B

2-4 1,571 feet

2-5 A

2-6 A

2-7 $\frac{x}{y}$

2-8 A

2-9 a represents the exponents for the base 2 shown in b; 1, $\frac{1}{2}$, $\frac{1}{4}$

3-1 8

3-2 E

3-3 $2^3 \times 107$

3-4 a) 3, 107
b) 2, 3, 6, 8, 11, 24, 33, 44, 88, 132
c) 2, 3, 6, 83, 166, 299

3-5 D

3-6 8 minutes 45 seconds; 7 laps; 5 laps

3-7 47 is a prime number so it has no common factors other than 1 with the other values.

4-1 $n = 40$

4-2 C

4-3 C

4-4 Martin

4-5 D

4-6 C

4-7 No; the dimensions have unequal ratios; 14.4 feet; 0.55 feet; model would be $\frac{5}{27}$ of the original height.

5-1 D

5-4 measure pairs of opposite sides, measure angles as equal or not equal to 90°; measure sides, measure angles as equal or not equal to 90°; no entry, measure angles as equal or not equal to 90°; measure distance between, no entry or draw a transversal and measure opposite angles, etc.

5-5 B

5-6 No; sides opposite congruent angles are congruent; No; third angle is 90°; isosceles right triangle

5-7 B

6-1 A

6-2 D

6-3 A

6-4 25% if 8 by 8 by 10 or 95% if 10 by 10 by 10

6-5 36 inches

6-6 D

6-7 A

6-8 If you measure to the nearest foot, then no growth is shown. If you measure to the nearest inch, then growth is shown. If you measure to the nearest half inch, then a more accurate measure of growth is shown. If you measure to the nearest quarter inch, then it will be more difficult to measure accurately. So to the nearest half inch is probably the best for the accuracy needed.

7-1 No; the sides are not equal.

7-2 B

7-3 C

7-4 No; $9 + 25 \neq 49$

7-5 D

7-6 about 0.25 inches

7-7 C

8-1 B

8-2 B

8-3 B

8-4 NP, P, P, NP

8-5 A

8-6 D

8-7 Incorrectly used subtraction to conclude that both grew by 3,000; Town A grew by 60% and Town B grew by 50% or $\frac{3}{5} > \frac{1}{2}$.

8-8 $.26

9-1 C

9-2 6; 2

9-3 $\frac{3}{8}$

9-4 beef-milk; beef-juice; tuna-milk; tuna-juice; ham-milk; ham-juice

9-5 C

9-6 No; 25% because there are four possible outcomes: WW, WB, BW, BB

9-7 16 men

10-1 $P = 2b + 2(a + 3)$ or $P = 2(a + b + 3)$

10-2 any number > 18

10-3 $n = 20$

10-4 B

10-5 B

10-6 (-2, 2), (-4, 4), (0, 6), (1, 8) (2, 10)

10-7 $14; 5 hours

10-8 $x = 7$

10-9 E

11-1 B

11-2 subtract 2.5

11-3 29

11-4 $222

11-5 B

11-6 $y = 2x - 1$

11-7 422; to make the nth figure, make an n by n square then add 2 blocks or make a $n + 1$ square and subtract $2n - 1$ blocks

12-1 A

12-2 (-2, 2); (-1, 5); (0, 8); (1, 11); (2, 14)

12-3 25 blinks per minute

12-4 B

12-5 Yes; if the person earns $60,000.

12-6 $633,333 in sales

BEST COPY AVAILABLE

Acknowledgments

Initial drafts of this publication were prepared by David Nohara, curriculum consultant, and by the SREB panel for mathematics: Pam Adamson, formerly of the Georgia Department of Education, Atlanta, Georgia.; Henry Cooke, South Point High School, South Point, Ohio; Dianne Hardy, Habersham High School, Clarksville, Georgia.; Katherine Mosely, Booker T. Washington High School, Atlanta, Georgia.; Linda Patch, North Carolina Department of Public Instruction, Raleigh, North Carolina; and Kathleen Carpenter, SREB.

Gene Bottoms, Caro Feagin, Sondra Cooney and Mary Myers of SREB also contributed to this publication.

The publication is supported by funds from the Office of Educational Research and Improvement, U.S. Department of Education; the Edna McConnell Clark Foundation; The Goldman-Sachs Foundation; the Carnegie Corporation of New York; and the Wallace-Reader's Digest Funds. The opinions expressed here do not necessarily reflect the positions or policies of any of the funding entities, and no official endorsement should be inferred.

Southern Regional Education Board Goals for Education

1. All children are ready for the first grade.
2. Achievement in the early grades for all groups of students exceeds national averages and performance gaps are closed.
3. Achievement in the middle grades for all groups of students exceeds national averages and performance gaps are closed.
4. All young adults have a high school diploma — or, if not, pass the GED tests.
5. All recent high school graduates have solid academic preparation and are ready for postsecondary education and a career.
6. Adults who are not high school graduates participate in literacy and job-skills training and further education.
7. The percentage of adults who earn postsecondary degrees or technical certificates exceeds national averages.
8. Every school has higher student performance and meets state academic standards for all students each year.
9. Every school has leadership that results in improved student performance — and leadership begins with an effective school principal.
10. Every student is taught by qualified teachers.
11. The quality of colleges and universities is regularly assessed and funding is targeted to quality, efficiency and state needs.
12. The state places a high priority on an education system of schools, colleges and universities that is accountable.

BEST COPY AVAILABLE

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

NOTICE

Reproduction Basis

X	This document is covered by a signed "Reproduction Release (Blanket)" form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.
	This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").